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Minimisation of active power losses and number of control adjustments in the optimal reactive dispatch problem

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Abstract: Trade-offs between power system's optimal operational performance and the minimal number of control adjustments necessary to attain a desired operating point make optimal reactive dispatch (ORD) solutions practical to system operators. In this study, a multi-objective ORD model that provides, in terms of weighting factors, trade-offs between minimal active power losses in transmission systems and minimal number of control adjustments in generator voltages, tap ratios and shunt controls is featured. This multi-objective ORD is formulated as a mixed-integer non-linear programming (MINLP) problem, and the proposed resolution methodology is based on translating the original MINLP problem into non-linear programming (NLP) problem deploying a sigmoid function, enabling the use of NLP solvers. Both original MINLP and translated NLP models are implemented in GAMS and numerical tests with IEEE test-systems with up to 300 buses are conducted using DICOPT, KNITRO and CONOPT solvers to validate the proposed ORD model and its resolution methodology. Results demonstrate the relation between active power losses and the number of adjustments in control variables, which is valuable information for operation planning. Another fundamental result is the high computational performance of the method when compared to specialized MINLP solvers.

1 Introduction

Optimal power flows (OPFs) have gained special attention due to novel challenges in transmission system operation caused by increasing penetration of renewable energy sources such as wind and solar, new high-voltage DC systems and the management of an aging infrastructure. In view of these characteristics, the consensus is to operate the system in an optimised mode while guaranteeing its robustness, flexibility and economic efficiency.

Since the seminal work done by Carpentier [1], which consisted in a new formulation for the economic dispatch problem by the incorporation of power flow equations into the set of equality constraints, many improvements on modelling and resolution methodologies have been proposed [2–6]. Moreover, the development of robust solvers for such a complex class of problems has been shown as a timely research topic [7, 8].

Many improvements have focused on the mathematical optimisation of standard and simplified formulations expressed in continuous non-linear programming (NLP) form [9–11], neglecting relevant practical aspects for power system operators. Only a few studies consider the integer modelling of some decision variables, mainly due to the increase in problem complexity and computational time for its resolution, especially for large systems [12]. On the other hand, for practical aspects, a small number of control adjustments to implement over a time period to attain a desired operational performance is preferred. Standard OPF formulations usually have limited practical scope in this sense, since optimisation techniques adjust the decision variables to attain the optimum. Those aspects are mentioned in [13–15], where the review on the state of the art of OPF formulation, resolution methods and critical analysis on the deficiencies in OPF modelling that restrains its practical use for power system operation are presented. One of the deficiencies pointed out in these works and that still imposes difficulties in the practical use of optimal reactive dispatch (ORD) solutions concerns the number of control adjustments used to attain the power system's optimal operational performance, since the number of control adjustments may be too large to be implemented in a short time interval, whether in normal or post-contingency operating scenarios. In fact, this deficiency may be considered as one of the major drawbacks in the practical use of OPF solutions.

Few studies have focused on limiting the number of adjustments of control variables in OPF models. In [16], the minimisation of active power losses for the IEEE 30-bus test-system is solved in two stages: in the first stage, the OPF is solved by a sequential linear programming technique; then, in the second stage, an expert-system-based optimisation technique is used to alleviate constraint violations, minimising the number of switching actions associated with control adjustments. Another approach found in the literature is to empirically limit the adjustment of control variables. In this context, two approaches to the OPF problem formulation which sets a predetermined number of control adjustments were proposed in [17].

The main difficulty in limiting the number of control adjustments is associated with the fact that control variables participate in both improving the objective function and satisfying operational constraints. The precise evaluation of the impact of the joint application of these controls is not trivial [14, 15]. Therefore, the trade-offs between a power system's optimal operation performance while satisfying operational constraints and the minimal number of control adjustments necessary to attain such an operating point is desirable [18].

There is a conflicting relation between minimising active power losses and minimising the number of control adjustments in transmission systems. Considering the published papers in which the main contributions are either the numerical resolution of large non-convex NLP problems or the modelling of integer variables, the main contribution of this paper is the proposal of a multi-objective ORD model designed to be solved efficiently. The system's performance is represented by a non-linear, non-convex and non-separable objective function and binary variables are used to model whether control variables are adjusted or not. To solve the proposed ORD, the corresponding mixed-integer NLP (MINLP) problem is translated into an NLP problem, converting the binary variables into continuous by means of a sigmoid function, which enables the use of NLP solvers. To validate such a methodology, the translated NLP problem is implemented in GAMS and solved by a commercial solver (CONOPT), whose results are compared to the ones obtained by the resolution of the original MINLP problem by two specialised solvers (DICOPT and KNITRO). Furthermore, the two conflicting objective functions are analysed in a multi-objective perspective considering different weighting factors.

In summary, this paper handles the numerical optimisation of large non-convex MINLP problems, overcoming additional difficulties arisen from the optimisation of multiple conflicting objectives with different formulation domains (continuous versus integer).

The remaining of this paper is structured as follows: Section 2 presents the multi-objective ORD problem to minimise active power losses and the number of control adjustments. In Section 3, the proposed methodology for solving the proposed multi-objective ORD problem is presented. Numerical tests carried out with IEEE benchmark test-systems with up to 300 buses are shown in Section 4. Finally, the main contributions of the paper are summarised and highlighted in Section 5.

2 Multi-objective ORD

Initially, two ORD problems, which compose the proposed multi-objective model, will be presented separately. They are the minimisation of active losses in power transmission systems and the minimisation of control adjustments necessary to attain the desired performance.

2.1 ORD model to minimise active power losses

The minimisation of active losses in transmission power systems is formulated as follows:

$$\begin{aligned}
\min \quad & P_{\text{Loss}} \\
\text{s.t.:} \quad & P_k - \sum_{m \in \mathscr{V}_k} P_{km}(\boldsymbol{V}, \boldsymbol{\theta}, \boldsymbol{t}) = 0, \quad \forall k \in \mathscr{G}' \cup \mathscr{C} \\
& Q_k + Q_k^{\text{sh}}(V_k) - \sum_{m \in \mathscr{V}_k} Q_{km}(\boldsymbol{V}, \boldsymbol{\theta}, \boldsymbol{t}, \boldsymbol{b}^{\text{sh}}) = 0, \quad \forall k \in \mathscr{C} \\
& Q_{G_k}^{\min} \le Q_{G_k}(\boldsymbol{V}, \boldsymbol{\theta}, \boldsymbol{t}, \boldsymbol{b}^{\text{sh}}) \le Q_{G_k}^{\max}, \quad \forall k \in \mathscr{G} \\
& V_k^{\min} \le V_k \le V_k^{\max}, \quad \forall k \in \mathscr{B} \\
& t_{km}^{\min} \le t_{km} \le t_{km}^{\max}, \quad \forall k, m \in \mathscr{T} \\
& b_k^{\text{sh}^{\min}} \le b_k^{\text{sh}} \le b_k^{\text{sh}^{\max}}, \quad \forall k \in \mathscr{B}^{\text{sh}}
\end{aligned} \tag{1}$$

where P_{Loss} is the total active power loss on transmission components

$$P_{\text{Loss}} = \sum_{k, m \in \mathscr{L} \cup \mathscr{T}} g_{km}\left(\frac{1}{t_{km}^2} V_k^2 + V_m^2 - 2\frac{1}{t_{km}} V_k V_m \cos\theta_{km}\right), \tag{2}$$

and $\boldsymbol{V}$ and $\boldsymbol{\theta}$ are, respectively, the voltage magnitude and phase angle vectors on generation and load buses; $\boldsymbol{t}$ is the vector of tap ratio of on-load tap-changer (OLTC) in-phase transformers; $\boldsymbol{b}^{\text{sh}}$ is the vector of shunt susceptances associated with capacitor banks and reactors; P_k and Q_k are, respectively, the active and reactive power injections at bus k; P_{km} and Q_{km} are, respectively, the active and reactive power flows from bus k to bus m; Q_{G_k} is the reactive power output of generators or synchronous condensers connected to bus k; Q_k^{sh} is the reactive power injection at bus k by shunt capacitor banks or shunt reactors; $t_{km}^{\min}$ and $t_{km}^{\max}$ are the vectors with lower and upper bounds of the tap ratio of OLTC in-phase transformers, respectively; $b_k^{\text{sh}^{\min}}$ and $b_k^{\text{sh}^{\max}}$ are the vectors with lower and upper bounds of the shunt susceptances associated with capacitor banks and reactors, respectively; g_{km} is the series conductance of branch $k - m$; and $\theta_{km} = \theta_k - \theta_m$. In (1), $\boldsymbol{t}$ and $\boldsymbol{b}^{\text{sh}}$ are modelled as continuous variables, however, from a practical standpoint, these control variables can only be adjusted by discrete steps.

In addition, $\mathscr{B}$ is the set of all system buses; $\mathscr{G}$ is the set of all generation buses; $\mathscr{G}'$ is the set of all generation buses except for the slack bus, $\mathscr{C}$ is the set of load buses; $\mathscr{L}$ is the set of all transmission line branches; $\mathscr{T}$ is the set of all in-phase OLTC transformers, $\mathscr{B}^{\text{sh}}$ is the set of all shunt capacitor banks and reactors, $\mathscr{V}_k$ is the set of index of buses connected to bus k through transmission lines and transformers.

2.2 ORD model to minimise the number of control adjustments

The minimisation of control adjustments necessary to attain the desired performance can be formulated by the incorporation of appropriate constraints and bounds on control adjustments [14]. The resulting formulation of this ORD problem is

$$\begin{aligned}
\min \quad & N \\
\text{s.t.:} \quad & P_k - \sum_{m \in \mathscr{V}_k} P_{km}(\boldsymbol{V}, \boldsymbol{\theta}, \boldsymbol{t}) = 0, \quad \forall k \in \mathscr{G}' \cup \mathscr{C} \\
& Q_k + Q_k^{\text{sh}}(V_k) - \sum_{m \in \mathscr{V}_k} Q_{km}(\boldsymbol{V}, \boldsymbol{\theta}, \boldsymbol{t}, \boldsymbol{b}^{\text{sh}}) = 0, \quad \forall k \in \mathscr{C} \\
& Q_{G_k}^{\min} \le Q_{G_k}(\boldsymbol{V}, \boldsymbol{\theta}, \boldsymbol{t}, \boldsymbol{b}^{\text{sh}}) \le Q_{G_k}^{\max}, \quad \forall k \in \mathscr{G}, \\
& V_k^{\min} \le V_k \le V_k^{\max}, \quad \forall k \in \mathscr{B} \\
& \boldsymbol{t}_{km}^{\min} \le t_{km} \le \boldsymbol{t}_{km}^{\max}, \quad \forall k, m \in \mathscr{T} \\
& \boldsymbol{b}_k^{\text{sh}^{\min}} \le b_k^{\text{sh}} \le \boldsymbol{b}_k^{\text{sh}^{\max}}, \quad \forall k \in \mathscr{B}^{\text{sh}} \\
& P_{\text{Loss}} \le P_{\text{Loss}}^{\max}, \\
& s_{1_k}(V_k^{\min} - V_k^0) \le V_k - V_k^0, \quad \forall k \in \mathscr{G} \\
& V_k - V_k^0 \le s_{1_k}(V_k^{\max} - V_k^0), \quad \forall k \in \mathscr{G} \\
& s_{2_{km}}(\boldsymbol{t}_{km}^{\min} - t_{km}^0) \le t_{km} - t_{km}^0, \quad \forall k, m \in \mathscr{T} \\
& t_{km} - t_{km}^0 \le s_{2_{km}}(\boldsymbol{t}_{km}^{\max} - t_{km}^0), \quad \forall k, m \in \mathscr{T} \\
& s_{3_k}\left(\boldsymbol{b}_k^{\text{sh}^{\min}} - b_k^{\text{sh}^0}\right) \le b_k^{\text{sh}} - b_k^{\text{sh}^0}, \quad \forall k \in \mathscr{B}^{\text{sh}} \\
& b_k^{\text{sh}} - b_k^{\text{sh}^0} \le s_{3_k}\left(\boldsymbol{b}_k^{\text{sh}^{\max}} - b_k^{\text{sh}^0}\right), \quad \forall k \in \mathscr{B}^{\text{sh}} \\
& s_1, s_2, s_3 \in \{0; 1\}
\end{aligned} \tag{3}$$

where N is the number of control adjustments

$$N = \sum_{k \in \mathscr{G}} s_{1_k} + \sum_{k, m \in \mathscr{T}} s_{2_{km}} + \sum_{k \in \mathscr{B}^{\text{sh}}} s_{3_k}, \tag{4}$$

and $P_{\text{Loss}}^{\max}$ corresponds to the maximum active power losses used to represent a desired system performance; V_k^0 is the initial value assigned to V_k; t_{km}^0 is the initial value assigned to t_{km}; $b_k^{\text{sh}^0}$ is the initial value assigned to b_k^{sh}; and s_{1_k}, $s_{2_{km}}$ and s_{3_k} are binary variables associated with, respectively, control adjustments in V_k, t_{km} and b_k^{sh}; variables s_{1_k}, $s_{2_{km}}$ and s_{3_k} assume 1 when the control variable must be adjusted and 0 when it must not. Therefore, the controls considered in the model (3) are the voltage magnitude set of generators and synchronous condensers (i.e. voltage control on generation buses), tap ratios of OLTC transformers and equivalent susceptance of capacitor banks and reactors.

2.3 Proposed multi-objective ORD model

The multi-objective model proposed in this paper aims at minimising active losses as well as the number of control adjustments. Such a combination of conflicting objectives and the handling of integer variables in the following model increases the complexity of this ORD problem. Thus, the proposed multi-objective ORD problem is formulated as follows:

$$
\begin{aligned}
\min \quad & [P_{\text{Loss}}, N]^{\text{T}} \\
\text{s.t.}: \quad & P_k - \sum_{m \in \mathscr{V}_k} P_{km}(\boldsymbol{V}, \boldsymbol{\theta}, \boldsymbol{t}) = 0, \quad \forall k \in \mathscr{G}' \cup \mathscr{C} \\
& Q_k + Q_k^{\text{sh}}(V_k) - \sum_{m \in \mathscr{V}_k} Q_{km}(\boldsymbol{V}, \boldsymbol{\theta}, \boldsymbol{t}, \boldsymbol{b}^{\text{sh}}) = 0, \quad \forall k \in \mathscr{C} \\
& Q_{G_k}^{\min} \le Q_{G_k}(\boldsymbol{V}, \boldsymbol{\theta}, \boldsymbol{t}, \boldsymbol{b}^{\text{sh}}) \le Q_{G_k}^{\max}, \quad \forall k \in \mathscr{G} \\
& V_k^{\min} \le V_k \le V_k^{\max}, \quad \forall k \in \mathscr{B} \\
& \boldsymbol{t}_{km}^{\min} \le t_{km} \le \boldsymbol{t}_{km}^{\max}, \quad \forall k, m \in \mathscr{T} \\
& \boldsymbol{b}_k^{\text{sh}^{\min}} \le b_k^{\text{sh}} \le \boldsymbol{b}_k^{\text{sh}^{\max}}, \quad \forall k \in \mathscr{B}^{\text{sh}} \\
& s_{1_k}(V_k^{\min} - V_k^0) \le V_k - V_k^0, \quad \forall k \in \mathscr{G} \\
& V_k - V_k^0 \le s_{1_k}(V_k^{\max} - V_k^0), \quad \forall k \in \mathscr{G} \\
& s_{2_{km}}(\boldsymbol{t}_{km}^{\min} - t_{km}^0) \le t_{km} - t_{km}^0, \quad \forall k, m \in \mathscr{T} \\
& t_{km} - t_{km}^0 \le s_{2_{km}}(\boldsymbol{t}_{km}^{\max} - t_{km}^0), \quad \forall k, m \in \mathscr{T} \\
& s_{3_k}\left(\boldsymbol{b}_k^{\text{sh}^{\min}} - b_k^{\text{sh}^0}\right) \le b_k^{\text{sh}} - b_k^{\text{sh}^0}, \quad \forall k \in \mathscr{B}^{\text{sh}} \\
& b_k^{\text{sh}} - b_k^{\text{sh}^0} \le s_{3_k}\left(\boldsymbol{b}_k^{\text{sh}^{\max}} - b_k^{\text{sh}^0}\right), \quad \forall k \in \mathscr{B}^{\text{sh}} \\
& s_{1_k} \in \{0, 1\} k \in \mathscr{G} \\
& s_{2_{km}} \in \{0, 1\} k, m \in \mathscr{T} \\
& s_{3_k} \in \{0, 1\} k \in B^{\text{sh}}
\end{aligned} \tag{5}
$$

For the sake of simplicity, the mathematical formulation of (5) may be restated in the compact general form

$$
\begin{aligned}
\min \quad & [f(x), S]^{\text{T}} \\
\text{s.t.}: \quad & g_i(x) = 0, \quad i = 1, \dots, p \\
& h_i(x) \le 0, \quad i = 1, \dots, q \\
& x_i^{\min} \le x_i \le x_i^{\max}, \quad i = 1, \dots, n \\
& s_i(x_{1_i}^{\min} - x_{1_i}^0) \le x_{1_i} - x_{1_i}^0, \quad i = 1, \dots, m_1 \\
& x_{1_i} - x_{1_i}^0 \le s_i(x_{1_i}^{\max} - x_{1_i}^0), \quad i = 1, \dots, m_1 \\
& s_i \in \{0; 1\}, \quad i = 1, \dots, m_1
\end{aligned} \tag{6}
$$

where S is the number of control adjustments with $S = \sum_{i=1}^{m_1} s_i$; $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2)$, $\boldsymbol{x}_1 \in \mathbb{R}^{m_1}$ is the vector of continuous control variables (since $\boldsymbol{t}$ and $\boldsymbol{b}^{\text{sh}}$ are modelled as continuous control variables in (5)) and $\boldsymbol{x}_2 \in \mathbb{R}^{m_2}$ is the vector of dependent variables; $\boldsymbol{x} \in \mathbb{R}^n$; $\boldsymbol{x}^{\min}, \boldsymbol{x}^{\max} \in \mathbb{R}^n$ are vectors with lower and upper bounds of x, respectively; $\boldsymbol{x}_1^{\min}, \boldsymbol{x}_1^{\max} \in \mathbb{R}^{m_1}$ are vectors with lower and upper bounds of $\boldsymbol{x}_1$; $\boldsymbol{x}_1^0$ is the vector of initial values assigned to $\boldsymbol{x}_1$; $\boldsymbol{s}$ is the vector of binary variables related to each variable of $\boldsymbol{x}_1$ (s_i assumes 1 when the variable x_{1_i} must be adjusted and 0 when it must not); $f: \mathbb{R}^n \mapsto \mathbb{R}$; $g: \mathbb{R}^n \mapsto \mathbb{R}^p$, with $p < n$; and $h: \mathbb{R}^n \mapsto \mathbb{R}^q$.

3 Strategies for solving the proposed multi-objective ORD problem

3.1 Weighting method

In the weighting method, the idea is to associate each objective function with a weighting coefficient and minimise the weighted summation of objectives. In this sense, the multi-objective problem is converted into a single objective problem in terms of weighting factors, and classical optimisation methods may be used to solve the problem. The information on this approach and its optimality can be found in [19].

Now consider the following multi-objective optimisation problem:

$$
\begin{aligned}
\min \quad & F(x) = [f_1(x), \dots, f_k(x)]^{\text{T}} \\
\text{s.t.}: \quad & g_i(x) = 0, \quad i = 1, \dots, p \\
& h_i(x) \le 0, \quad i = 1, \dots, q \\
& x_i^{\min} \le x_i \le x_i^{\max}, \quad i = 1, \dots, n
\end{aligned} \tag{7}
$$

where $k \ge 2$ represents the number of objective functions in $F: R^n \mapsto R^k$.

Thus, the multi-objective optimisation problem (7) is transformed into the following weighting problem:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{k} \omega_i f_i(x) \\
\text{s.t.}: \quad & g_i(x) = 0, \quad i = 1, \dots, p \\
& h_i(x) \le 0, \quad i = 1, \dots, q \\
& x_i^{\min} \le x_i \le x_i^{\max}, \quad i = 1, \dots, n
\end{aligned} \tag{8}
$$

where $\omega_i \ge 0$ is the weighting factor associated with the ith objective function in $F(x)$. ω_i is a positive real number such that the summation of all ω_i is greater than zero. Generally, it is assumed that $\sum_{i=1}^{k} \omega_i = 1$.

Therefore, the proposed multi-objective ORD (6) can be formulated as the following problem:

$$
\begin{aligned}
\min \quad & \omega_1 f(x) + \omega_2 S \\
\text{s.t.}: \quad & g_i(x) = 0, \quad i = 1, \dots, p \\
& h_i(x) \le 0, \quad i = 1, \dots, q \\
& x_i^{\min} \le x_i \le x_i^{\max}, \quad i = 1, \dots, n \\
& s_i(x_{1_i}^{\min} - x_{1_i}^0) \le x_{1_i} - x_{1_i}^0, \quad i = 1, \dots, m_1 \\
& x_{1_i} - x_{1_i}^0 \le s_i(x_{1_i}^{\max} - x_{1_i}^0), \quad i = 1, \dots, m_1 \\
& s_i \in \{0; 1\}, \quad i = 1, \dots, m_1
\end{aligned} \tag{9}
$$

Notice that for $k = 2$, ω_1 can be exchanged by ω and ω_2 by $(1 - \omega)$ so that there is only one varying weighting factor, whose summation remains equal to 1.

3.2 Sigmoid-function-based multi-objective model for handling binary variables

There are many real life MINLP problems whose integer variables are modelled as binary variables. However, depending on the system size and complexity, few exact methods can efficiently solve such problems successfully. This happens due to the complexity of handling a large number of binary variables. For example, even one of the simplest cases with a quadratic objective function and linear constraints is classified as an NP-hard problem [20].

In order to deal with binary variables in problems such as (9), a heuristic strategy that employs the sigmoid function proposed in [21], which was originally applied to transmission expansion planning problems, is used here. To deal with binary variables in (9), the sigmoid function $\chi: \mathbb{R} \mapsto \mathbb{R}$ is defined as

$$
\chi(y) = \frac{e^{\tau y} - 1}{e^{\tau y} + 1}, \tag{10}
$$

where τ is the slope of the sigmoid function. The higher the value assigned to τ, the greater the slope of the sigmoid function, which tends to a right angle. The argument y is a non-negative real value since the sigmoid function χ assumes negative values between -1 and 0 for $y < 0$.

The inclusion of (10) into (9) consists in substituting variables s by $\chi(y)$, which results in the following modified problem:

Given (9), define χ for each binary variable in s;
Build (11);
Set $k = 0$;
Set $x_1^0, x_2^0, y^0, \tau^0$ and define c, ξ_1;
while (12) *is not satisfied* **do**
  Solve (11) for a fixed value of τ^k;
  Save $(x_1^k, x_2^k, \chi(y^k))$ as the current solution;
  $\tau^{k+1} = c\tau^k$;
  $k = k + 1$;
end

Fig. 1 *Algorithm 1: Proposed algorithm for solving MINLP problems with binary variables*

Table 1 Characteristics of the ORD problem for each IEEE system

Test system	Equality constraints	Continuous variables	Control variables
IEEE 14	22	31	9
IEEE 30	53	65	12
IEEE 57	106	222	25
IEEE 118	181	430	77
IEEE 300	530	1142	189

$$
\begin{aligned}
\min \quad & \omega_1 f(x) + \omega_2 \sum_{i=1}^{m_1} \chi(y_i) \\
\text{s.t.:} \quad & g_i(x) = 0, \quad i = 1, \ldots, p \\
& h_i(x) \le 0, \quad i = 1, \ldots, q \\
& x_i^{\min} \le x_i \le x_i^{\max}, \quad i = 1, \ldots, n \\
& \chi(y_i)(x_{1_i}^{\min} - x_{1_i}^0) \le x_{1_i} - x_{1_i}^0, \quad i = 1, \ldots, m_1 \\
& x_{1_i} - x_{1_i}^0 \le \chi(y_i)(x_{1_i}^{\max} - x_{1_i}^0), \quad i = 1, \ldots, m_1 \\
& 0 \le y_i \le y_i^{\max}, \quad i = 1, \ldots, m_1
\end{aligned} \tag{11}
$$

since χ is a function that returns a real value between 0 and 1 as τ has its value increased, i.e. $\chi(y_i)$ is used to model the binary variable s_i such that the assumption $\chi(y_i) = s_i$ is valid and, thus, the modified problem (11) associated with (9) can be solved by the NLP algorithms.

3.3 Proposed algorithm

The proposed algorithm consists in solving a sequence of modified problems such as (11) for increasing values of τ until all y_i, $i = 1, \ldots, m_1$, assume binary values in χ under a certain numerical tolerance. The variable τ is increased linearly by the factor c, which is set within the half-closed interval (1,10] (the end point 1 is not included in such an interval because τ would never increase if c could assume the value 1).

By successively increasing τ, a sequence of NLP problems is, then, solved and, as a result, the binary variables modelled by the sigmoid function will converge to binary values. However, the successive increase in τ may cause ill-conditioning issues. For this situation, a heuristic strategy to improve the convergence of the algorithm by reducing ill-conditioning issues has been devised.

The process of successive resolutions of (11) and parameter setting τ continues until a convergence criterion is satisfied. The convergence criterion consists in assessing whether $\chi(y_i)$ in the kth iteration assumes a binary value. Mathematically, such a criterion is represented by

$$\| \chi(y^k) - \boldsymbol{o}^k \|_\infty \le \xi_1 \tag{12}$$

where $\boldsymbol{o}^k$ is the vector of binary elements (0 or 1), with each element i of $\boldsymbol{o}^k$ corresponding to the nearest binary value of each element i of $\chi(y^k)$; and ξ_1 is the algorithm's convergence tolerance.

The proposed algorithm for an MINLP problem with binary variables as (9) is summarised in Algorithm 1 (see Fig. 1).

According to [21], while the only value that maps $\chi(y)$ to 0 is 0, the upper-bound argument of the sigmoid function may take any value that maps $\chi(y)$ close to 1. Many tests have been carried out to determine which value of y maps $\chi(y)$ sufficiently close to 1, and the upper-bound argument that has provided the best results is $y = 20$. Therefore, the variable y should be in the range $0 \le y \le 20$, in which the upper bound has been determined empirically. To speed up the algorithm's convergence, $\chi(y)$ is rounded-off when it is sufficiently close to 0 or 1. So, if $\chi(y_i^k) \le 0.1$, we consider $\chi(y_i^k) = 0$; on the other hand, if $\chi(y_i^k) \ge 0.9$, we consider $\chi(y_i^k) = 1$. The resolution complexity of this algorithm depends on the complexity of each corresponding NLP. Therefore, different solvers with different algorithms may also present different performances.

4 Numerical results and analysis

The multi-objective ORD models represented by (9) and (11) have been implemented in GAMS modelling language [22]. For the MINLP problem (9), DICOPT [23] and KNITRO [24] solvers were used. Both DICOPT and KNITRO are MINLP commercial solvers for optimisation problems with binary variables. DICOPT is based on three key ideas: outer approximation, equality relaxation and augmented penalty. The MINLP algorithm used by DICOPT solves a series of NLP and mixed-integer programming (MIP) sub-problems using any available NLP or MIP solver in GAMS. In the current simulation, the NLP sub-problems have been solved by CONOPT [25, 26] (which applies the generalised reduced gradient algorithm) and MIP sub-problems have been solved by CPLEX [27] by DICOPT. The KNITRO MIP code offers two algorithms for MINLP. The first one is a nonlinear branch-and-bound method and the second one uses the hybrid Quesada-Grossman method [28] for convex MINLP. The model (11), which represents our proposal, has been solved with CONOPT solver.

The test systems used to verify the robustness and efficacy of the proposed ORD model and resolution methodology are the IEEE 14, 30, 57, 118 and 300-bus benchmark test-systems (Table 1). The simulations have been carried out on a PC with i7-4770 @ 3,4 GHz processor running MS Windows 10 operating system with 8 GB of RAM.

For all systems, voltage magnitude limits $V_k^{\min}$ and $V_k^{\max}$ for each bus k were 0.90 and 1.10 p.u.; tap ratio limits $\boldsymbol{t}_{km}^{\min}$ and $\boldsymbol{t}_{km}^{\max}$ for each OLTC transformer were 0.88 and 1.12 p.u.; and $\boldsymbol{b}_k^{\mathrm{sh}^{\min}}$ and $\boldsymbol{b}_k^{\mathrm{sh}^{\max}}$ limits were determined for each system and can be found in [29]. In all tests with the proposed Algorithm 1 (Fig. 1), the convergence tolerance ξ_1 was 10^{-4}.

Tables 2–7 summarise the results of the simulations for the IEEE 14, 30, 57, 118 and 300-bus test-systems, respectively; the simulation considered the weight ω varying from 0 to 1 with steps of 0.1. In order to verify the existence of feasible solutions, smaller steps have also been tested. For instance the IEEE 118-bus test-system, smaller steps have been used to find solutions to fill the gap between the weights (Table 6). These steps have been chosen heuristically.

Solutions with the symbol 'a' in Tables 2–7 are dominated by others, i.e. dominated solutions are those that can be rejected because a better solution has been found in any of the objectives for other values of ω. For instance, in Table 5, consider the results for DICOPT: for $\omega = 0.1$ and $\omega = 0.2$ the number of control adjustments is the same ($N = 7$), but the active losses are better for $\omega = 0.1$. Thus, the solution found for $\omega = 0.2$ is dominated by the solution found for $\omega = 0.1$. We can also use the term dominated solutions for results obtained from different solvers. In Table 4, all solvers found solutions with $N = 1$, but KNITRO found the best solution for $N = 1$ when active power losses are taken into account. Thus, we can say that the solution for $N = 1$ found by KNITRO dominates the solutions found by DICOPT and the sigmoid function with CONOPT.

Table 2 Simulation results for each solver for the IEEE 14-bus test-system

ω	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
					DICOPT						
N	0	0	0	2	4	4	4	5	4	6	9
losses, MW	13.39	13.39	13.39	12.88	12.34	12.34	12.34	12.31	12.34	12.28	12.27
ΔN	—	0	0	2	2	0	0	1	−1	2	3
ΔLosses, MW	—	0	0	−0.51	−0.54	0	0	−0.03	0.03	−0.06	−0.01
time, s	0.250	0.228	0.405	0.727	0.529	0.490	0.839	0.488	0.509	0.521	0.416
					KNITRO						
N	0	0	0	2	4	4	5	5	5	7	9
losses, MW	13.39	13.39	13.39	12.88	12.34	12.34	12.31	12.31	12.31	12.27	12.27
ΔN	—	0	0	2	2	0	1	0	0	2	1
ΔLosses, MW	—	0	0	−0.51	−0.54	0	−0.03	0	0	−0.04	0
time, s	0.307	0.210	0.299	0.306	0.306	0.300	0.306	0.300	0.307	0.420	0.298
					Sigmoid function and CONOPT						
N	0	0	0	2	4	4	4	4	4	5	8
losses, MW	13.39	13.39	13.39	12.88	12.34	12.34	12.34	12.34	12.34	12.31	12.28
ΔN	—	0	0	2	2	0	0	0	0	1	3
ΔLosses, MW	—	0	0	−0.51	−0.54	0	0	0	0	−0.03	−0.03
iterations	2	2	2	2	2	2	2	3	3	3	3
time, s	0.242	0.213	0.204	0.350	0.389	0.337	0.371	0.349	0.356	0.336	0.397

N is the number of control adjustments and losses is the active power losses found for the corresponding weighting factor ω_i, respectively; $\Delta N = N^{(\omega_i)} - N^{(\omega_i - 1)}$;
$\Delta\text{Losses} = \text{Losses}^{(\omega_i)} - \text{Losses}^{(\omega_i - 1)}$; time is the elapsed time for the resolution given by the solver.

Table 3 Simulation results for each solver for the IEEE 30-bus test-system

ω	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
					DICOPT						
N	0	0	1	2	6	4	4	5	7	9	11
losses, MW	17.55	17.55	17.54	16.91	16.11	16.32	16.32	16.25	16.09	15.98	15.98
ΔN	—	0.00	1.00	1.00	4.00	−2.00	0.00	1.00	2.00	2.00	2.00
ΔLosses, MW	—	0.00	−0.01	−0.63	−0.80	0.21	0.00	−0.07	−0.16	−0.11	0.00
time, s	0.246	0.406	0.423	0.746	0.567	0.955	0.835	0.828	1.102	1.214	0.854
					KNITRO						
N	0	0	0	2	4	4	6	4	7	9	11
losses, MW	17.55	17.55	17.55	16.91	16.32	16.32	16.09	16.32	16.04	15.99	16.00
ΔN	—	0.00	0.00	2.00	2.00	0.00	2.00	−2.00	3.00	2.00	2.00
ΔLosses, MW	—	0.00	0.00	−0.64	−0.59	0.00	−0.23	0.23	−0.28	−0.05	0.01
time, s	0.305	0.307	0.303	0.417	0.419	0.412	0.409	0.442	0.533	0.532	0.306
					Sigmoid function and CONOPT						
N	0	0	0	4	4	6	7	7	8	9	11
losses, MW	17.55	17.55	17.55	16.32	16.32	16.09	16.04	16.04	16.01	15.98	16.00
ΔN	—	0.00	0.00	4.00	0.00	2.00	1.00	0.00	1.00	1.00	2.00
ΔLosses, MW	—	0.00	0.00	−1.23	0.00	−0.23	−0.05	0.00	−0.03	−0.03	0.02
iterations	2	2	2	4	4	4	4	4	4	4	3
time, s	0.228	0.241	0.213	0.624	0.593	0.582	0.626	0.580	0.581	0.586	0.397

N is the number of control adjustments and losses is the active power losses found for the corresponding weighting factor ω_i, respectively; $\Delta N = N^{(\omega_i)} - N^{(\omega_i - 1)}$;
Δ Losses = $\text{Losses}^{(\omega_i)} - \text{Losses}^{(\omega_i - 1)}$; time is the elapsed time for the resolution given by the solver.

Fig. 2 illustrates the results shown in Table 4. The losses and the number of control adjustments are those found for the multi-objective models (9) and (11) for the IEEE 57-bus system. After removing the dominated solutions among the models, it results in Fig. 3. Similarly, Fig. 4 presents the multi-objective curve originated from the combination of the solutions for the IEEE 118-bus system. Fig. 5 depicts the results of the proposed method for the IEEE 300-bus system. The figure has been generated for (11), which considers the weighting method combined with the sigmoid function for handling binary variables. In this latter case, only the proposed method was successful to find the solutions.

4.1 Analysis of the results

The solutions for the extreme values $\omega = 0$ (minimisation of control adjustment) and $\omega = 1$ (minimisation of active power losses) cannot be considered as solutions of the multi-objective. However, these cases are taken into account as bounds for the number of control adjustments and the active power losses.

The solutions of problem (9) obtained by solvers DICOPT and KNITRO have been compared with the solutions obtained by SIGMOID-CONOPT for the modified problem (11) and it is possible to confirm the high equivalence among them.

Observing the results of the IEEE 14 and 30-bus systems, DICOPT and KNITRO solvers were able to find good solutions for all weighting factors. Considering the IEEE 57-bus system (Table 4), DICOPT and CONOPT provided seven non-dominated solutions, while KNITRO provided nine different non-dominated solutions. Comparing solution by solution and disregarding dominated solutions among solvers, DICOPT contributes with four solutions to the combined curve, while KNITRO has eight and CONOPT has six. This result is depicted in Fig. 3. Some solvers

Table 4 Simulation results for each solver for the IEEE 57-bus test-system

ω	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
					DICOPT						
N	1	5	8	8	9	9	8	14	11	17[a]	24
losses, MW	26.04	24.74	21.86	21.86	21.75	21.75	21.86	21.19	21.67	21.49	20.92
ΔN	—	4.00	3.00	0.00	1.00	0.00	−1.00	6.00	−3.00	6.00	7.00
ΔLosses, MW	—	−1.30	−2.88	0.00	−0.11	0.00	0.11	−0.67	0.48	−0.18	−0.57
time, s	0.281	1.078	1.411	1.756	0.865	1.265	0.932	2.940	1.780	2.013	0.946
					KNITRO						
N	2[a]	2	1	6	8	9	11	14	15	18	24[a]
losses, MW	26.18	25.47	25.83	24.09	21.86	21.85	21.40	21.15	21.07	20.97	21.37
ΔN	—	0.00	−1.00	5.00	2.00	1.00	2.00	3.00	1.00	3.00	6.00
ΔLosses, MW	—	−0.71	0.36	−1.74	−2.23	−0.01	−0.45	−0.25	−0.08	−0.10	0.40
time, s	0.344	0.852	0.440	0.855	0.880	1.186	2.489	3.252	5.108	7.077	0.413
					Sigmoid function and CONOPT						
N	1[a]	1	1	8	8	8	11	15	16	19	25
losses, MW	26.16	26.04	26.04	21.86	21.86	21.86	21.40	21.07	21.01	20.96	20.92
ΔN	—	0.00	0.00	7.00	0.00	0.00	3.00	4.00	1.00	3.00	6.00
ΔLosses, MW	—	−0.12	0.00	−4.18	0.00	0.00	−0.46	−0.33	−0.06	−0.05	−0.04
iterations	5	5	5	4	4	4	4	4	4	4	5
time, s	0.590	0.577	0.727	0.648	0.630	0.616	0.536	0.529	0.606	0.496	0.621

N is the number of control adjustments and losses is the active power losses found for the corresponding weighting factor ω_i, respectively; $\Delta N = N^{(\omega_i)} - N^{(\omega_{i-1})}$;
$\Delta\text{Losses} = \text{Losses}^{(\omega_i)} - \text{Losses}^{(\omega_{i-1})}$; time is the elapsed time for the resolution given by the solver.
[a]Solutions are dominated by others.

Table 5 Simulation results for each solver for the IEEE 118-bus test-system

ω	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
					DICOPT						
N	13[a]	7	7[a]	12	12[a]	50	55	57	57[a]	61	75
losses, MW	132.26	131.35	131.80	129.85	131.53	115.75	110.52	106.72	110.46	106.39	106.05
ΔN	—	−6.00	0.00	5.00	0.00	38.00	5.00	2.00	0.00	4.00	14.00
ΔLosses, MW	—	−0.91	0.45	−1.95	1.68	−15.78	−5.23	−3.80	3.74	−4.07	−0.34
time, s	7.016	2.787	22.643	166.511	28.820	276.049	42.849	15.803	7.518	24.183	1.216
					KNITRO						
N	1	1[a]	2	5	—	—	57	60	59	60	76
losses, MW	132.46	132.48	131.94	130.32	—	—	106.57	106.23	106.37	106.23	106.05
ΔN	—	0.00	1.00	3.00	—	—	—	3.00	−1.00	1.00	16.00
ΔLosses, MW	—	0.02	−0.54	−1.62	—	—	—	−0.34	0.14	−0.14	−0.18
time, s	2.772	7.734	128.324	234.619	1000.002	1000.005	51.137	41.812	27.535	42.295	0.629
					Sigmoid function and CONOPT						
N	1	1	1	5	15	55	58	59	60	62	70
losses, MW	132.48	132.48	132.48	128.88	125.12	107.82	106.38	106.25	106.20	106.12	106.05
ΔN	—	0.00	0.00	4.00	10.00	40.00	3.00	1.00	1.00	2.00	8.00
ΔLosses, MW	—	0.00	0.00	−3.60	−3.76	−17.30	−1.44	−0.13	−0.05	−0.08	−0.07
iterations	7	7	7	3	3	3	3	3	3	3	3
time, s	0.932	1.162	0.873	0.557	1.010	1.776	2.320	1.982	2.310	2.311	2.060

N is the number of control adjustments and losses is the active power losses found for the corresponding weighting factor ω_i, respectively; $\Delta N = N^{(\omega_i)} - N^{(\omega_{i-1})}$;
$\Delta\,\text{Losses} = \text{Losses}^{(\omega_i)} - \text{Losses}^{(\omega_{i-1})}$; time is the elapsed time for the resolution given by the solver.
[a]Solutions are dominated by others.

presented the same points, e.g. 8, 11 and 15 control actions. Only the symbol representing the fastest solver is presented in the figure.

The difficulty in solving MINLP problems with binary variables by MINLP solvers can be noticed from the results obtained by DICOPT and KNITRO. As the problem size increases, the number of continuous and binary variables also increases and the drop on the efficiency of MINLP solvers can be noticed from the increase on the computation time and failure on the convergence for the IEEE 118-bus system.

For the IEEE 118-bus system, tests with other values of ω have been necessary; while such an analysis for the other systems used 11 values of ω, the analysis for the IEEE 118-bus system was carried out using 24 values of ω (Tables 5 and 6). DICOPT and KNITRO found, respectively, 13 and 8 non-dominated solutions, while CONOPT found 16 non-dominated solutions. Comparing solution by solution and disregarding dominated solutions among solvers, DICOPT has no solution in the combined curve, while KNITRO has 4 and CONOPT has 11. This result is depicted in Fig. 4. Concerning the quality of the results, DICOPT provides low performance feasible solutions, while KNITRO failed to provide feasible solutions for five values of ω. The proposed resolution methodology using the sigmoid function and CONOPT found feasible solutions for all weighting factors, and it is competitive with DICOPT and KNITRO. Another important characteristic of the sigmoid function and CONOPT is its computational processing time; the proposed resolution methodology using the sigmoid

Table 6 Simulation results for each solver for the IEEE 118-bus test-system considering smaller steps for weighting factor ω between 0.4 and 0.5

ω	0.41	0.42	0.43	0.44	0.441	0.442	0.443	0.444	0.445	0.446	0.447	0.448	0.449
						DICOPT							
N	14[a]	17	27	37	35	35	35	35	35	35	35	13	28
losses, MW	131.79	129.86	128.01	122.61	122.92	122.92	122.92	122.92	122.92	122.92	122.92	131.14	124.29
ΔN	—	3	10	10	−2	0	0	0	0	0	0	−22	15
ΔLosses, MW	—	−1.93	−1.85	−5.40	0.31	0.00	0.00	0.00	0.00	0.00	0.00	8.22	−6.85
time, s	734.155	166.167	28.469	220.242	181.058	189.893	224.332	807.792	271.753	229.948	261.315	303.988	219.617
						KNITRO							
N	13[a]	59[a]	—	12[a]	12	12[a]	—	63[a]	63[a]	63[a]	—	13[a]	13[a]
losses, MW	126.78	124.81	—	120.18	119.74	130.11	—	127.69	120.87	118.36	—	126.22	122.08
ΔN	—	46	—	—	0	0	—	—	0	0	—	—	0
ΔLosses, MW	—	−1.97	—	—	−0.44	10.37	—	—	−6.82	−2.51	—	—	−4.14
time, s	1017.283	1000.114	1000.077	1000.096	1000.080	1000.094	1000.076	1000.082	1000.089	1000.090	1000.082	1000.084	1000.078
						Sigmoid function and CONOPT							
N	15	20	26	27	27	27	27	29	38	47	52	52	52
losses, MW	125.12	123.64	121.20	120.67	120.67	120.67	120.67	119.69	116.48	112.48	109.90	109.90	109.90
ΔN	—	5	6	1	0	0	0	2	9	9	5	0	0
ΔLosses, MW	—	−1.48	−2.44	−0.53	0.00	0.00	0.00	−0.98	−3.21	−4.00	−2.58	0.00	0.00
iterations	3	3	3	3	3	3	3	3	3	3	3	3	3
time, s	0.636	0.609	0.754	0.742	0.818	0.750	0.811	1.187	1.017	1.191	1.526	1.310	1.191

N is the number of control adjustments and losses is the active power losses found for the corresponding weighting factor ω_i, respectively; $\Delta N = N^{(\omega_i)} - N^{(\omega_{i-1})}$;
$\Delta\text{Losses} = \text{Losses}^{(\omega_i)} - \text{Losses}^{(\omega_{i-1})}$; time is the elapsed time for the resolution given by the solver.
[a]Solutions are dominated by others.

Table 7 Simulation results for sigmoid function and CONOPT for the IEEE 300-bus test-system

ω	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
						Sigmoid function and CONOPT					
N	0	3	29	38	57	73	85	94	105	116	180
losses, MW	408.19	397.27	369.23	361.40	351.40	345.79	343.64	342.29	341.47	341.04	340.78
ΔN	—	3.00	26.00	9.00	19.00	16.00	12.00	9.00	11.00	11.00	64.00
ΔLosses, MW	—	−10.92	−28.04	−7.83	−10.00	−5.61	−2.15	−1.35	−0.82	−0.43	−0.26
iterations	2	2	3	3	3	3	3	3	3	3	3
time, s	0.442	0.987	1.897	2.570	3.717	5.634	5.781	6.422	6.001	7.097	7.930

N is the number of control adjustments and losses is the active power losses found for the corresponding weighting factor ω_i, respectively; $\Delta N = N^{(\omega_i)} - N^{(\omega_{i-1})}$;
$\Delta\,\text{Losses} = \text{Losses}^{(\omega_i)} - \text{Losses}^{(\omega_{i-1})}$; time is the elapsed time for the resolution given by the solver.

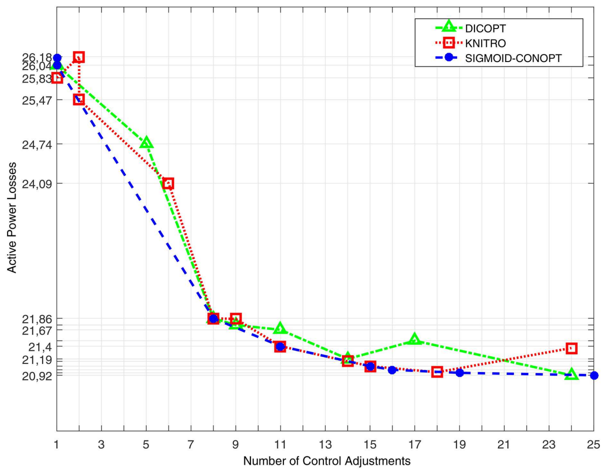

Fig. 2 *Active power losses by the number of control adjustments for the IEEE 57-bus system (obtained with ω varying from 0 to 1)*

function and CONOPT demanded, in average, 29.824 s to find all 24 solutions, while DICOPT demanded 4434.124 s and KNITRO 15555.190 s to run all cases.

For the IEEE 300-bus test-system (Table 7), the proposed method successfully provides results for ω varying from 0 to 1 in steps of 0.1 using the sigmoid function and CONOPT in <50 s. On the other hand, both MINLP solvers, i.e. KNITRO and DICOPT, failed in this task.

5 Conclusions

In this paper, a multi-objective ORD model which aims at minimising the active power losses on the transmission system and the number of control adjustments has been proposed. The main idea is to find the compromise between the minimisation of losses and the required number of control actions, providing more options for system operation. In order to make the ORD study feasible taking into account the complexity of the problem which originally consists in an MINLP problem, we have proposed a transformed NLP model that uses a sigmoid function for handling with binary variables which are solved by an iterative algorithm. The result of the method is feasible for the original MINLP model.

The method has been tested with the IEEE benchmark test-systems. The results have indicated good performance of the proposed resolution methodology in comparison to the specialised MINLP solvers (DICOPT and KNITRO). When testing with small systems, such as the IEEE 57-bus system, DICOPT and KNITRO provided good non-dominated solutions, and the combination of all methods allowed the construction of the two objectives curve which enriches the final solution. However, the curse of dimensionality has been observed for larger systems such as the IEEE-118 and IEEE 300-bus systems, where DICOPT and KNITRO have presented, in some cases, elevated computation time to obtain feasible results or even stagnation, while the

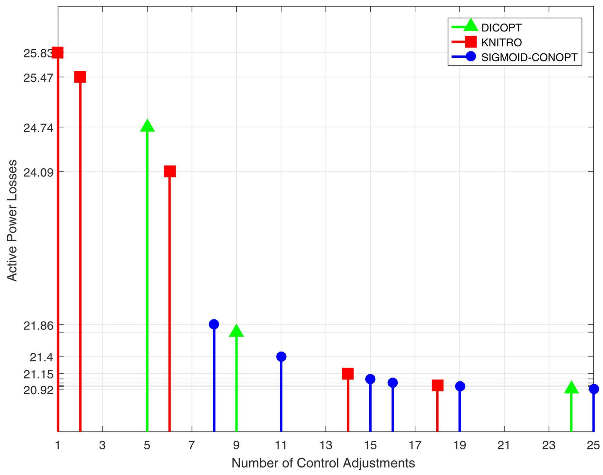

Fig. 3 *Active power losses by the number of control adjustments for the IEEE 57-bus test-system: non-dominated solutions obtained for the proposed ORD problem combining three different solvers*

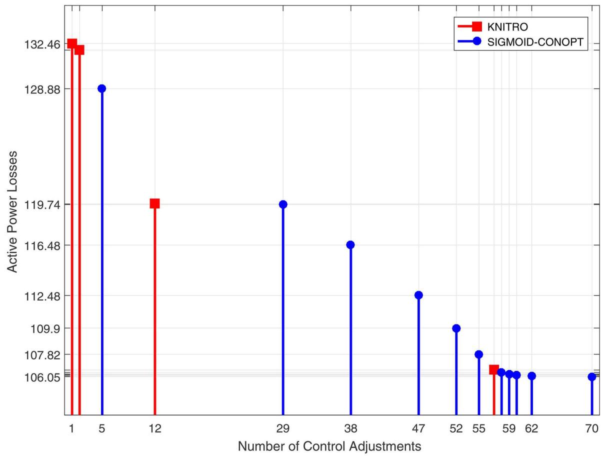

Fig. 4 *Active power losses by the number of control adjustments for the IEEE 118-bus test-system: non-dominated solutions obtained for the proposed ORD model combining three different solvers*

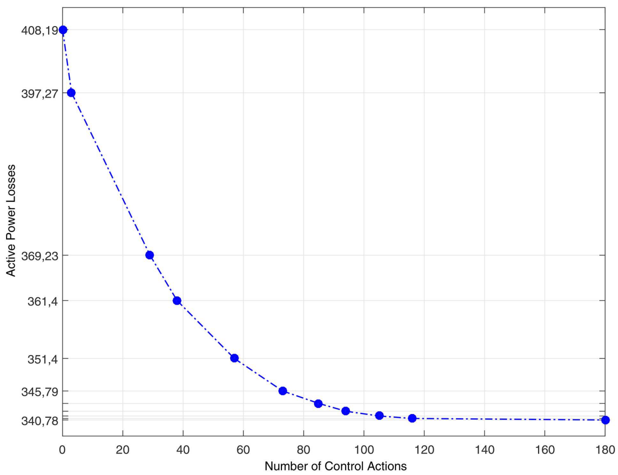

Fig. 5 *Active power losses by the number of control adjustments for the IEEE 300-bus system (obtained with ω varying from 0 to 1), using the sigmoid function for handling binary variables*

proposed method SIGMOID-CONOPT succeeded with feasible solutions and low processing time. We also believe the proposed sigmoid function-based approach has potential to be applied to a multi-horizon framework. Care must be taken in this case if time coupling constraints exist.

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